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Entanglement of a General Formalism V-Type Three-Level Atom Interacting with a Single-Mode Field in the Presence of Nonlinearities

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Abstract We investigate the evolution of the atomic quantum entropy and the atom-field entanglement in a system of a V-configuration three-level atom interacting with a single-mode field with additional forms of nonlinearities of both the field and the intensity-dependent atom-field coupling. With the derivation of the unitary operator within the frame of the dressed state and the exact results for the state of the system we perform a careful investigation of the temporal evolution of the entropy. A factorization of the initial density operator is assumed, considering the field to be initially in a squeezed coherent or binomial state. The effects of the mean photon number, detuning, Kerr-like medium and the intensity-dependent coupling functional on the entropy are analyzed.

Keywords Entanglement · Quantum entropy

1 Introduction

It is important to note that, a great deal of activity has centered on the analysis of the physical properties of nonlinear interaction models describing a localized center coupled to the modes of a quantized bosonic field [1, 2]. There are many physical situations where such models may find applications [3, 4]. For example, it may be of interest in the context of the effective Hamiltonian approach to the two-mode two-photon micromaser [5, 6]. Moreover, it is worthwhile to remark that investigating such models goes beyond an intrinsic theoretical interest in condensed matter systems too, because the development of new and improved materials is expected to lead to the fabrication of three dimensional photonic band gap systems possessing few isolated high-Q resonant field modes [7–9]. The physical origin of these nonlinear interactions may be traced back to the existence of a strong coupling between few levels of the material center and some selected modes of the quantized elastic or electromagnetic field. Very often, the problem of interest is how to investigate the effects of nonlinearity on the quantum dynamics of the system starting from an appropriately chosen

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initial conditions. The starting point is the construction of an effective basic Hamiltonian model, which contains the essential ingredients of the microscopic physical situation, and at the same time provide us with an exactly solvable model. In this paper, we consider the introduction of such Hamiltonian.

In recent years much attention has been focused on the properties of the entanglement between the field and the atom and, in particular, the entropy of the Jaynes-Cummings system [10–15]. The authors in [10, 11] have shown that von Neumann entropy is a very useful operational measure of the purity of the quantum state, which automatically includes all moments of the density operator. The time evolution of the field (atomic) entropy reflects the time evolution of the degree of entanglement between the atom and the field. The higher the entropy, the greater the entanglement. An expression for the field entropy for the entangled state of a single two-level atom interacting with a single electromagnetic field mode in an ideal cavity with the atom undergoing either a one or a two-photon transition has been studied [15]. Furthermore, the evolution of the (atomic) field entropy for the three-level atom one-mode[16, 17] and two-mode[18–20] model has been studied.

Here, we aim at extending the previously cited treatments to study the problem of a three-level atom in V-configuration interacting with a single-mode field including acceptable kinds of nonlinearities of both the field and the intensity-dependent atom-field coupling. Our goal is to investigate the properties of the entanglement of the above-mentioned system, particularly we examine the influences of the mean photon number, the input field distribution, the field nonlinearity (Kerr-like medium), detuning and the different intensity-dependent coupling functionals on the degree of entanglement.

The organization of this paper is as follows. In Sect. 2 we introduce our Hamiltonian model and give an exact expression for the unitary operator $U(t)$ in the frame of the dressed state formalism. In Sect. 3 we employ the analytical results obtained in Sect. 2 to investigate the properties of the degree of entanglement due to the von Neumann entropy. We devote Sect. 4 to our discussion. Finally we give our conclusion in Sect. 5.

2 The Model

The Hamiltonian of the system in the rotating-wave approximation is of the form ($\hbar = 1$)

$$H = H_0 + H_{in} \tag{1}$$

$$H_0 = \sum_{j=1}^{3} \omega_j \sigma_{j,j} + \Omega \hat{a}^+ \hat{a} \tag{2}$$

where ω_1, ω_2 and ω_3 are the atomic levels energies ($\omega_1 > \omega_2 > \omega_3$), and Ω is the field frequency, with the detuning parameters Δ_1 and Δ_2 given by $\Delta_1 = -\Omega + (\omega_1 - \omega_3)$ and $\Delta_2 = -\Omega + (\omega_2 - \omega_3)$. The operators $\hat{a}$ and $\hat{a}^+$ are the boson operators for the field satisfying $[\hat{a}, \hat{a}^+] = 1$. The operators σ_{ij} represent the atomic projection operators and satisfy the following commutation relations $[\sigma_{ij}, \sigma_{kl}] = \sigma_{il}\delta_{jk} - \sigma_{kj}\delta_{il}$, while $[\hat{a}, \sigma_{ij}] = 0$, $(i, j = 1, 2, 3)$. The interaction part of the Hamiltonian in the presence of an arbitrary nonlinear medium can be written as

$$\begin{aligned} H_{in} = {} & \Re(\hat{a}^\dagger \hat{a}) + \lambda_1(\sigma_{13} \otimes f_1(\hat{a}^\dagger \hat{a}) \otimes \hat{a} + \hat{a}^+ \otimes f_1(\hat{a}^\dagger \hat{a}) \otimes \sigma_{31}) \\ & + \lambda_2(\sigma_{23} \otimes f_2(\hat{a}^\dagger \hat{a}) \otimes \hat{a} + \hat{a}^+ \otimes f_2(\hat{a}^\dagger \hat{a}) \otimes \sigma_{32}). \end{aligned} \tag{3}$$

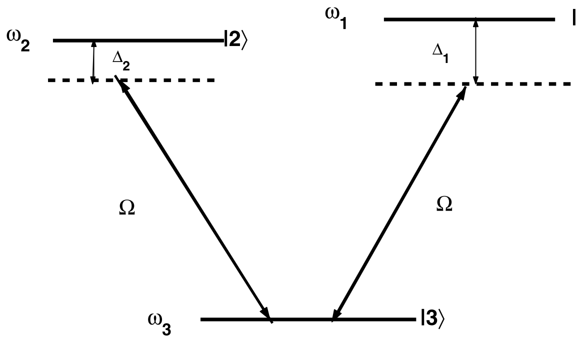

Fig. 1 Energy level diagram for a V-type three-level atom with a one-photon transition and detuning Δ_1, Δ_2

$\Re(\hat{a}^\dagger\hat{a})$ and $f(\hat{a}^\dagger\hat{a})$ are Hermitian operator functions of the photon number operator, such that $\lambda_1 f_1(\hat{a}^\dagger\hat{a})$ and $\lambda_2 f_2(\hat{a}^\dagger\hat{a})$ represent arbitrary intensity-dependent atom-field couplings, while $\Re(\hat{a}^\dagger\hat{a})$ denotes the one-mode field nonlinearity which can model Kerr-like medium nonlinearity as will be discussed later.

The initial state $|\Psi(0)_{AF}\rangle$ of the combined atom-field system may be written as

$$|\Psi(0)_{AF}\rangle = |\Psi(0)_A\rangle \otimes |\Psi(0)_F\rangle \tag{4}$$

where $|\Psi(0)_A\rangle = |1\rangle$ the initial state of the atom and $|\Psi(0)_F\rangle = |\Theta\rangle$ is the initial state of the field. The initial state $|\Theta\rangle = \sum p^{(n)}|n\rangle$ where the probability amplitude $p^{(n)}$ is defined in the usual manner as $p^{(n)} = \langle n|\Theta\rangle$.

The time evolution between the atom and the field is defined by the unitary evolution operator generated by H. Thus $U(t)$ is given by

$$U(t) \equiv \exp(-iHt).$$

This unitary operator $U(t)$ is written as

$$U(t) = \exp(-i\omega_3 t)|\Phi\rangle\langle\Phi| + \sum_{n=0}^{\infty}\sum_{j=1}^{3}\exp(-iE_j^{(n)}t)|\Psi_j^{(n)}(t)\rangle\langle\Psi_j^{(n)}(t)| \tag{5}$$

where the eigenvalues

$$E_j^{(n)} = -\frac{X_1}{3} + \frac{2}{3}\left(\sqrt{X_1^2 - 3X_2}\right)\cos(\theta_j) \tag{6}$$

and

$$\theta_j = \left(\frac{1}{3}\cos^{-1}\left[\frac{9X_1X_2 - 2X_1^3 - 27X_3}{2(X_1^2 - 3X_2)^{\frac{3}{2}}}\right] + (j-1)\frac{2\pi}{3}\right), \quad j = 1, 2, 3 \tag{7}$$

with

$$\begin{aligned}
X_1 &= -(r_1 + r_2 + r_3),\\
X_2 &= -\left[V_1^2 + V_2^2 - r_1r_2 - r_1r_3 - r_2r_3\right],\\
X_3 &= r_2V_1^2 + r_1V_2^2 - r_1r_2r_3,\\
r_1 &= \omega_1 + \Omega n + \Re(n),
\end{aligned} \tag{8}$$

$$
\begin{aligned}
r_2 &= \omega_2 + \Omega n + \Re(n), \\
r_3 &= \omega_3 + \Omega(n+1) + \Re(n+1), \\
V_1 &= \lambda_1 f_1(n)\sqrt{n+1}, \qquad V_2 = \lambda_2 f_2(n)\sqrt{n+1}
\end{aligned}
$$

and $|\Phi\rangle$, $|\Psi_j^{(n)}\rangle$ are the dressed states of the system associated with the eigenvalues ω_3 and $E_j^{(n)}$, $(j = 1, 2, 3)$

$$
\begin{aligned}
|\Phi\rangle &= |0, 3\rangle \\
|\Psi_j^{(n)}\rangle &= \alpha_j^{(n)}|n, 1\rangle + \beta_j^{(n)}|n, 2\rangle + \gamma_j^{(n)}|n+1, 3\rangle
\end{aligned}
\tag{9}
$$

where

$$
\begin{pmatrix} \alpha_j^{(n)} \\ \beta_j^{(n)} \\ \gamma_j^{(n)} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} -V_1(r_2 - E_j^{(n)}) \\ -V_2(r_1 - E_j^{(n)}) \\ (r_1 - E_j^{(n)})(r_2 - E_j^{(n)}) \end{pmatrix}
\tag{10}
$$

$$
M^2 = (r_1 - E_j^{(n)})^2(r_2 - E_j^{(n)})^2 + V_1^2(r_2 - E_j^{(n)})^2 + V_2^2(r_1 - E_j^{(n)})^2
\tag{11}
$$

Having obtained the explicit form of the unitary operator $U(t)$, the eigenvalues and the eigenfunctions for the system under consideration, we are therefore in a position to discuss the degree of entanglement (DEM) due to the von Neumann entropy of the system. This will be the subject of the following section.

3 Degree of Entanglement (DEM)

The physical essence of entanglement consists in the existence of quantum correlations between the individual parts of a composite system that have interacted once in the past but are no longer interacting. Formally, these correlations are caused by combination of the superposition principle in quantum mechanics with the tensor product structure of the space of states [21]. Quantum mechanically, the von Neumann entropy is defined as

$$
S = -\mathrm{Tr}\rho \ln \rho
\tag{12}
$$

where ρ is the density operator for a given quantum system and we have set Boltzmann's constant equal to 1. Quantum entropies are generally difficult to compute because they involve the diagonalization of large (and, in many cases, infinite dimensional) density matrices. If ρ describes a pure state, then $S = 0$, and if ρ describes a mixed state, then $S \neq 0$. Let S_A and S_F denote the entropies of two interacting systems (i.e A and F) and let S denote the entropy of the composite system. Araki and Lieb [22] showed that these entropies satisfy the triangle inequalities

$$
|S_A - S_F| \leq S \leq S_A + S_F
\tag{13}
$$

A nice illustration of these inequalities in the context of the Jaynes-Cummings model has been given by Knight and Phoenix [10, 11]. The entropies of the atom and the field, when treated as a separate system, are defined through the corresponding reduced density operators by

$$
S_{A(F)} = \mathrm{Tr}_{A(F)}(\rho_{A(F)} \ln \rho_{A(F)})
\tag{14}
$$

Here we use the quantum field entropy as a measure of the degree of entanglement between the field and the atom of the system under consideration. We assume that the system starts from a pure state. Consequently, its entropy S vanishes (*i.e.*, $S=0$). Therefore, inequality (13) implies that $S_A = S_F$. Since the trace is invariant under a similarity transformation, we can go to bases in which the atomic density matrix is diagonal and write (14) as.

$$\mathrm{DEM}(t) = S_A = S_F = -\sum_{j=1}^{3} \varrho_j \ln \varrho_j \tag{15}$$

Where ϱ_j is the eigenvalue for the atomic density matrix $\rho_A(t)$ with

$$\rho_A(t) = \mathrm{Tr}_F\,\rho(t) = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{21} & \rho_{22} & \rho_{23} \\ \rho_{31} & \rho_{32} & \rho_{33} \end{pmatrix} \tag{16}$$

In order to derive a calculation formalism of the entropy, we must obtain the system density operator, by means of which we obtain the reduced atomic density matrix and its eigenvalues. To achieve that we use (4) and (5) to obtain the final state of the system at any time $t>0$ as

$$\begin{aligned} \rho(t) &= U(t)|\Psi_{AF}(0)\rangle\langle\Psi_{AF}(0)|U^*(t) = \sum_{n,m=0}^{\infty} p^{(n)} p^{*(m)} U(t)|n,1\rangle\langle m,1|U^*(t) \\ &= \sum_{n,m=0}^{\infty} p^{(n)} p^{*(m)} \big(A(n,t)A^*(m,t)|n,1\rangle\langle m,1| + A(n,t)B^*(m,t)|n,1\rangle\langle m,2| \\ &\quad + A(n,t)C^*(m,t)|n,1\rangle\langle m+1,3| + B(n,t)A^*(m,t)|n,2\rangle\langle m,1| \\ &\quad + B(n,t)B^*(m,t)|n,2\rangle\langle m,2| + B(n,t)C^*(m,t)|n,2\rangle\langle m+1,3| \\ &\quad + C(n,t)A^*(m,t)|n+1,3\rangle\langle m,1| + C(n,t)B^*(m,t)|n+1,3\rangle\langle m,2| \\ &\quad + C(n,t)C^*(m,t)|n+1,3\rangle\langle m+1,3|\big) \end{aligned} \tag{17}$$

where

$$\begin{aligned} A(n,t) &= \sum_{j=1}^{3} \alpha_j^{*(n)} \alpha_j^{(n)} \exp\big(-itE_j^{(n)}\big) \\ B(n,t) &= \sum_{j=1}^{3} \alpha_j^{*(n)} \beta_j^{(n)} \exp\big(-itE_j^{(n)}\big) \\ C(n,t) &= \sum_{j=1}^{3} \alpha_j^{*(n)} \gamma_j^{(n)} \exp\big(-itE_j^{(n)}\big) \end{aligned} \tag{18}$$

with $\alpha_j^{(n)}$, $\beta_j^{(n)}$ and $\gamma_j^{(n)}$ given in (10) and $^{(*)}$ denotes the complex conjugate. Then the elements of atomic reduced density matrix $\rho_A(t)$ in (16) at any time t can be given as

$$\rho_{11} = \sum_{n=0} |p^{(n)}|^2 |A(n,t)|^2$$

$$\begin{aligned}
\rho_{22} &= \sum_{n=0} |p^{(n)}|^2 |B(n,t)|^2 \\
\rho_{33} &= \sum_{n=0} |p^{(n)}|^2 |C(n,t)|^2 \\
\rho_{12} &= \sum_{n=1} |p^{(n)}|^2 A(n,t) B^*(n,t) = \rho_{21}^* \\
\rho_{13} &= \sum_{n=1} p^{(n)} p^{*(n-1)} A(n,t) C^*(n-1,t) \\
\rho_{23} &= \sum_{n=0} p^{(n)} p^{*(n-1)} B(n,t) C^*(n-1,t) \\
\rho_{31} &= \sum_{n=0} p^{(n)} p^{*(n+1)} C(n,t) A^*(n+1,t) \\
\rho_{32} &= \sum_{n=0} p^{(n)} p^{*(n+1)} C(n,t) B^*(n+1,t)
\end{aligned} \tag{19}$$

Furthermore, the eigenvalues ϱ_j of the atomic density matrix $\rho_A(t)$ are the roots of the cubic equation

$$\varrho^3 + b_0 \varrho^2 + b_1 \varrho + b_2 = 0 \tag{20}$$

Where the coefficients b_0, b_1 and b_2 are given by the matrix elements of (16) as

$$\begin{aligned}
b_0 &= -(\rho_{11} + \rho_{22} + \rho_{33}) = -1 \\
b_1 &= \rho_{11}\rho_{22} + \rho_{11}\rho_{33} + \rho_{22}\rho_{33} - (\rho_{12}\rho_{21} + \rho_{13}\rho_{31} + \rho_{23}\rho_{32}) \\
b_2 &= \rho_{11}\rho_{23}\rho_{32} + \rho_{22}\rho_{13}\rho_{31} + \rho_{33}\rho_{12}\rho_{21} - (\rho_{11}\rho_{22}\rho_{33} + \rho_{13}\rho_{21}\rho_{32} + \rho_{12}\rho_{23}\rho_{31})
\end{aligned} \tag{21}$$

Therefore, the eigenvalues ϱ_j $(j = 1, 2, 3)$ are given by:

$$\varrho_j = \frac{1}{3} + \frac{2}{3}\left(\sqrt{1-3b_1}\right)\cos\left(\frac{1}{3}\cos^{-1}\left[\frac{2-9b_1-27b_2}{2(\sqrt{1-3b_1})^3}\right] + \frac{2\pi}{3}(j-1)\right) \tag{22}$$

Thus, according to (15) we obtain the degree of entanglement for the system under consideration. By using the above results, we discuss the dynamical behavior of the degree of entanglement of the present model. This will be done in the following section.

4 Results and Discussion

Based on the above observations, we present some interesting numerical results for different parameters to demonstrate the effect on the entanglement degree when the atom is initially prepared to be in its excited state and the field is being initially in (1) binomial state (2) squeezed coherent state. The binomial state $|\eta, M\rangle$ defined as a linear superposition of number states $|n\rangle$ in an M-dimensional subspace [23, 24] takes the form

$$|\eta, M\rangle = \sum_{n=0}^{M} p_n^M(\eta)|n\rangle, \tag{23}$$

where η is a complex number with $|\eta| \leq 1$, M is a positive integer and $p_n^M(\eta)$ the amplitude of the nth Fock state is given by

$$p_n^M(\eta) = \sqrt{\binom{M}{n}(\eta^n(1-|\eta|)^{M-n})}. \tag{24}$$

This state has $|\eta|M$ as its mean photon number $\bar{n}$ and the photon number variance $\mathrm{Var}(n) = (\Delta n)^2 = (1-|\eta|)\bar{n}$. As $\eta \to 0$, $M \to \infty$ such that $|\eta|M \to \bar{n}$, a fixed number, the state $|\eta, M\rangle$ tends to the coherent state $|\alpha = \sqrt{\eta M}\rangle$.

While, in the case of a squeezed coherent state, the photon number distribution is given by [25–27]

$$|p_n|^2 = \frac{(\tanh r)^n}{2^n n! \cosh r}\left|H_n\left(\frac{\varepsilon}{\sqrt{2\cosh r \sinh r}}\right)\right|^2 \exp\left[-|\varepsilon|^2 + \tanh r \quad \mathrm{Re}(\varepsilon)^2\right], \tag{25}$$

where, $\varepsilon = \alpha \cosh r + \alpha^* \sinh r$, $\alpha = |\alpha| \exp(i\varsigma)$ and H_n is the Hermite polynomial. Here, we suppose the minor axis of the ellipse, representing the direction of squeezing, parallel to the coordinate of the field oscillator. The initial phase ς of α is the angle between the direction of coherent excitation and the direction of squeezing. The mean photon number of this field is equal to $\bar{n} = |\alpha|^2 + \sinh^2 r$. When $r = 0$ one gets the photon distribution for the coherent state, whereas for $\alpha = 0$ the photon distribution of the squeezed vacuum state is recovered. The latter distribution is oscillatory with zeros for odd n.

Generally, we note that all considered cases have a common property namely, there are periodical changes always occurring in the atomic entropy. This should be expected, resulting from the existence of periodic functions in the expression of $S_A(t)$ see (22). In the case of a disentangled pure joint state $S_A(t)$ is zero, and for maximally entangled states it gives $\ln 3$. It does not appear possible to express the sums in (15) in closed form, but for not too large values of (the mean photon numbers), direct numerical evolutions can be performed. We point out that the mathematical sound truncation criteria have been invoked to compute all infinite series for the atomic wave function. Also with a great precision, an excellent accuracy for the behavior of the atomic entropy function $S_A(t)$ has been determined. In fact, as we mentioned before the entropy attains the zero value (i.e., disentanglement) when the atom is in one of its pure states (i.e., either in its upper or lower states) while strong entanglement occurs when the inversion is equal to zero. It is to be noted from the numerical calculations and figures that increasing the mean photon number leads to decreasing the entanglement degree (maximum values of entanglement). However the maximum value of the entanglement also varies and occurs for some short period of time. Also, the fluctuations decrease due to the increasing in the mean photon number. Finally, the maximum values of the atomic entropy decrease in the squeezed coherent case faster than the binomial case as the mean photon number increases. Hence, the maximum values of entanglement in the binomial case is larger than that for the squeezed coherent case (compare Figs. 2(1) and 2(2)).

In Fig. 3 we display the evolution of the atomic entropy for a various values of the detuning parameters (i.e., Δ_1, Δ_2). We note that the detuning parameter affects the maximum values of the entropy by bringing them down. This can be attributed to the fact that on having large detuning, the atomic system is weakly coupled to the field, and hence the degree of entanglement is weak (see Fig. 3). Furthermore, the minimum values of the entropy attain larger amounts as the detuning increases. Also, the period of collapses increase as soon as the value of the detuning parameters increase. While the period of collapses, the maximum (minimum) values of the atomic entropy and revival time not only depend on the values

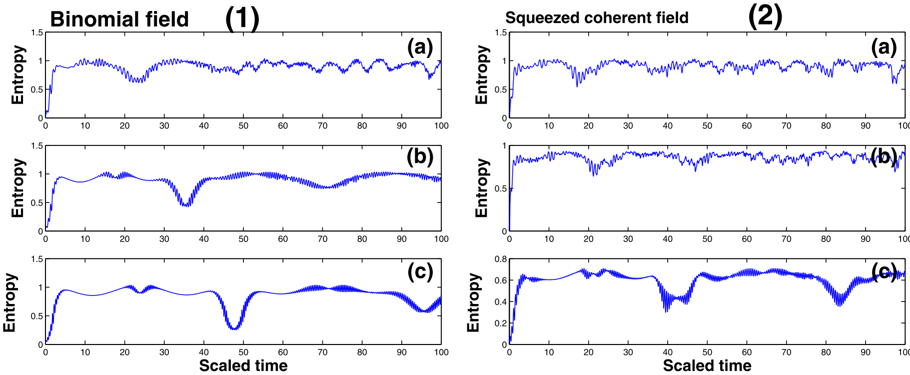

Fig. 2 Evolution of the DEM as function of the scaled time λt for an input (**1**) binomial field and (**2**) squeezed coherent field with $\lambda_1 = \lambda_2 = \lambda$, $\chi = 0$, $f_1(n) = f_2(n) = 1$, $\varsigma = 0$, $\Delta_1 = \Delta_2 = 0$ and for (**1**) (**a**) $\eta = 0.3$, $M = 20$, (**b**) $\eta = 0.5$, $M = 30$ and (**c**) $\eta = 0.7$, $M = 40$ while for (**2**)(**a**) $|\alpha| = \sqrt{3}$, $r = 1.3$, (**b**) $|\alpha| = \sqrt{5}$, $r = 1.7$ and (**c**) $|\alpha| = \sqrt{20}$, $r = 1.7$

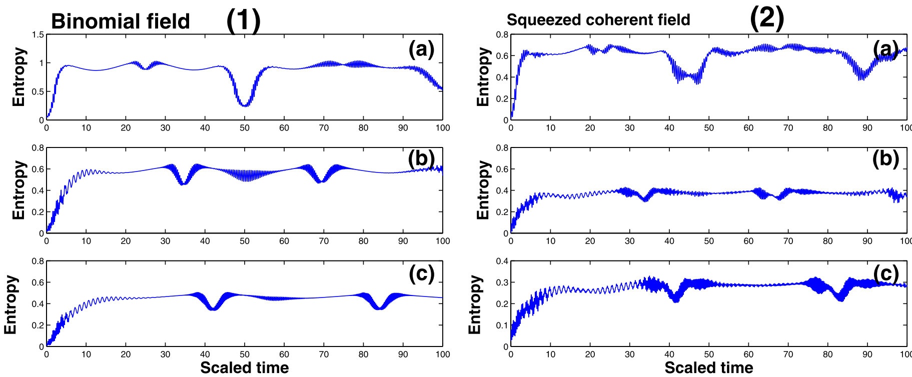

Fig. 3 Evolution of the DEM as function of the scaled time λt for an input (**1**) binomial field and (**2**) squeezed coherent field with $\lambda_1 = \lambda_2 = \lambda$, $\chi = 0$, $f_1(n) = f_2(n) = 1$, $\varsigma = 0$, and for (**1**) $\eta = 0.7$, $M = 40$, (**2**) $|\alpha| = \sqrt{20}$, $r = 1.7$ and for all (**a**) $\Delta_1 = 5$, $\Delta_2 = 4$, (**b**) $\Delta_1 = 15$, $\Delta_2 = 10$ and (**c**) $\Delta_1 = 20$, $\Delta_2 = 15$

of the detuning parameters but also on the input field distribution (compare Figs. 3(1) and Fig. 3(2)).

Figs. 4 illustrates the effect of the Kerr-like medium in the atomic entropy, we note that not only the maximum field entropy and the atom-field entanglement are reduced as the Kerr-like medium increases but also the collapse-revival phenomenon in the average photon number becomes less and less prominent as the Kerr medium increases. For a strong coupling strength of a Kerr-like medium the former starts to dominate the dynamics (there is nearly decoupling of the atom and field) and there is a reintroduction of some kind of regularity in the evolution of the system. This is quite apparent from the regular spikes present in Fig. 4. Also, the average photon number tends to become flat as there is an increased tendency for population trapping at such higher values of the Kerr-like nonlinearity. Finally, for small values of the Kerr-like medium, there is an increase of the sustainment time of the maximum field entropy, and strong entanglement of the field with the atom, while for large values, it results in a decrease of the field entropy, and the field is disentangled from the

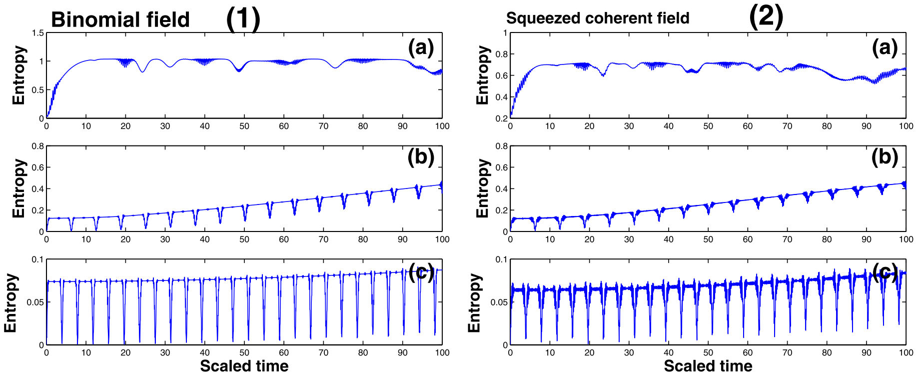

Fig. 4 Evolution of the DEM as functions of the scaled time λt for an input (**1**) binomial field and (**2**) squeezed coherent field with $\lambda_1 = \lambda_2 = \lambda$, $f_1(n) = f_2(n) = 1$, $\varsigma = 0$, $\Delta_1 = \Delta_2 = 0$ and for (**1**) $\eta = 0.7$, $M = 40$, (**2**) $|\alpha| = \sqrt{20}$, $r = 1.7$ and for all (**a**) $\chi = 0.1$, $(b)\chi = 0.5$ and $(c)\chi = 0.8$

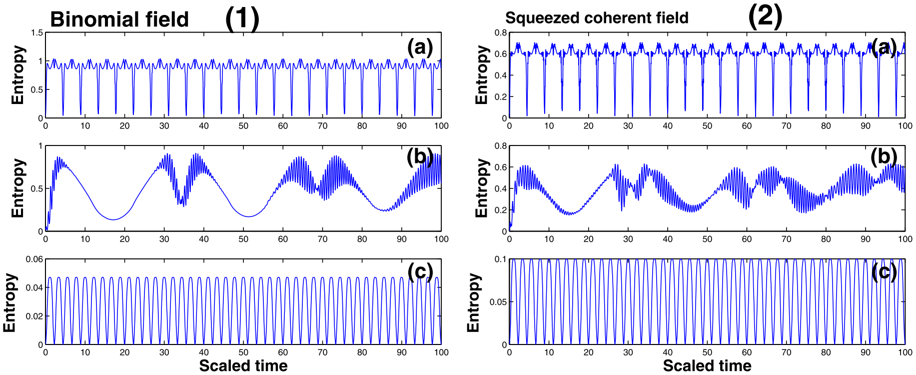

Fig. 5 Evolution of the DEM as function of the scaled time λt for an input (**1**) binomial field and (**2**) squeezed coherent field with $\lambda_1 = \lambda_2 = \lambda$, $\chi = 0$, $\varsigma = 0$, $\Delta_1 = \Delta_2 = 0$ for (**1**) $\eta = 0.7$, $M = 40$, (**2**) $|\alpha| = \sqrt{20}$, $r = 1.7$ and for all (**a**) $f_1(n) = f_2(n) = \sqrt{n+1}$, (**b**) $f_1(n) = 1$, $f_2(n) = \frac{1}{\sqrt{n+1}}$, (**c**) $f_1(n) = f_2(n) = \frac{1}{\sqrt{n+1}}$

atom during the time evolution. While the binomial input still has a larger maximum values of the entanglement than that for the squeezed coherent case even if the Kerr-like medium was added to the problem (compare Figs. 4(1) and 4(2)).

Now we will turn our attention to show the effect of different forms of the intensity-dependent coupling functional on the behavior of the atomic entropy the results are presents in Fig. 5, where the behavior of the atomic entropy changes strongly dependent on the used form of the intensity-dependent coupling functional. When $f_1(n) = f_2(n) = \sqrt{n+1}$ the Rabi frequency changes its dependence on $\sqrt{2(n+1)}$ for $f_1(n) = f_2(n) = 1$ to $\sqrt{2}(n+1)$, hence the stepwise excitation becomes larger than that for $f_1(n) = f_2(n) = 1$. So that the minimum values of the entanglement degree between the atom and the field must be decreased. Also, the pure state (disentanglement) may occur and the collapse and revival period is reduced as shown in Fig. 5(a). It is interesting to note that adding detuning or Kerr-like medium to this case increasing the minimum values of the atomic entropy and hence, the pure state no longer exists (compare Fig. 5(a) with Fig 6(a) and Fig. 7(a)).

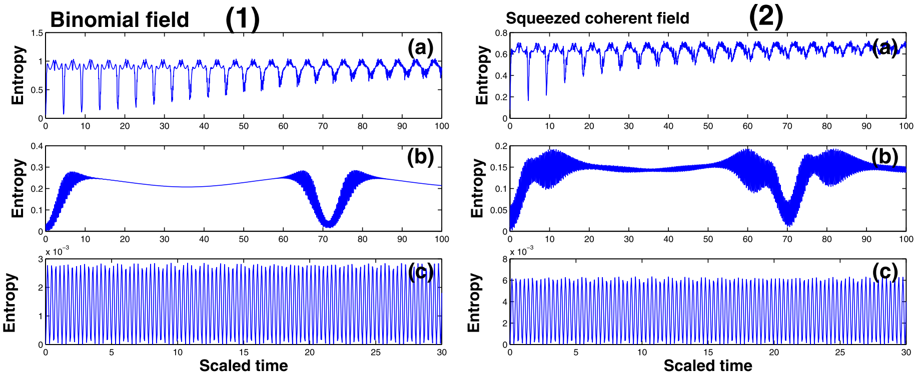

Fig. 6 Evolution of the DEM as function of the scaled time λt for an input (**1**) binomial field and (**2**) squeezed coherent field with the parameters as in Fig. 5, but $\Delta_1 = 20$ and $\Delta_2 = 15$

However, we notice that the detuning effect decreases the amplitude of the atomic entropy and increases the number of oscillation around the revival time (compare Fig. 5(a) with Fig. 6(a)). While Kerr adds some irregularity to the atomic entropy behavior and decreases the revival period as shown in Fig. 7(a). While, the situation is strongly changed when we take $f_1(n) = 1,\ f_2(n) = \frac{1}{\sqrt{n+1}}$ as shown in Fig. 5(b), where the first eye-catching property of the atomic entropy is the existence of the pronounced local minima appearing in the middle of the revival times. There also are the well-developed maxima at the times corresponding to the beginning of the revival regions. Furthermore, not only the revival (collapse) period becomes larger (smaller) than that for $f_1(n) = f_2(n) = 1$ but, also we note the maximum values of the entanglement degree is reduced here (compare Figs. 5(b) and 2(c)). This can be attributed to the smaller Rabi frequency in this case than that for $f_1(n) = f_2(n) = 1$. If we add detuning to this case, we note that the detuning parameters affect the maximum values of the atomic entropy by bringing them down. Also, the usual detuning effect in elongating the revival time and increasing the collapse period is obviously observed. While the minimum values of the atomic entropy attain larger amounts than that in the absence of detuning (compare Figs. 5(b) and 6(b)). On the other hand, if Kerr-like medium is adds as in Fig. 7(b), in this case, there are sharp peaks observed with some kind of periodicity where at the first stage of time the atomic entropy attains, its minimum values and then rises to the maximum values during the time evolution. Also, the collapse-revival phenomenon in the average photon number becomes lesser than that when the Kerr medium is absent (compare Figs. 5(b) and 7(b)).

But, due to the smallest Rabi frequency and its independence on n when $f_1(n) = f_2(n) = \frac{1}{\sqrt{n+1}}$ the evolution of the atomic entropy is quite interesting where in this case the entropy oscillates periodically between zero and its maximum values as shown in Fig. 5(c). On the other hand, by comparing this case with the case considered in Fig. 6(c) where the detuning is added to the problem, we note that adding detuning, increases the maximum values of the atomic entropy and large number of oscillations emerges on the evolution of the atomic entropy. Also, the pure state occurs more faster than that when the detuning is absent (compare Figs. 5(c) and 6(c)). But, there is a new phenomena appearing when the Kerr-like medium is added to the case in which $f_1(n) = f_2(n) = \frac{1}{\sqrt{n+1}}$, where the results are presented in Fig 7(c). In this case we note that the atom and the field disentanglement at the first stage of time and the atomic entropy has a zero value, then go up to the maximum values during the time evolution while, the Rabi oscillations completely disappear (compare

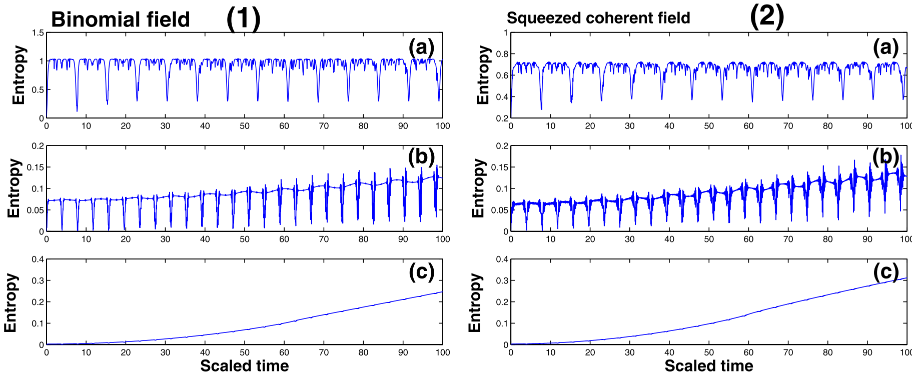

Fig. 7 Evolution of the DEM as function of the scaled time λt for an input (**1**) binomial field and (**2**) squeezed coherent field with the parameters as in Fig. 5, but $\chi = 0.8$

Figs. 5(c) and 7(c)). Finally, the degree of entanglement between the atom and the field not only it can be controlled by choosing forms of intensity-dependent coupling functional but also, it depends on the statistical properties of the input field distributions besides the detuning and Kerr-like medium parameters.

5 Conclusion

We have investigated the evolution of the atomic entropy and the atom-field entanglement, in the V-configuration of the three-level atom, interacting with a single-mode field in a cavity. We take explicitly into account the existence of forms of nonlinearities of both the field and the intensity-dependent atom-field coupling when the input field being initially in either a binomial or squeezed coherent states. The exact results are employed to perform a careful investigation of the mentioned problems. The effects of the mean photon number, detuning, Kerr-like medium and the intensity-dependent coupling functional on the investigated problems are analyzed. The general conclusions reached and illustrated by numerical results show that

- Generally, the evolution of the atomic entropy and the atom-field entanglement depend on the input field distributions.
- Increasing the mean photon number leads to decreasing the degree of entanglement (maximum values of entanglement).
- The degree of entanglement between the atom and the field can be controlled by choosing the right forms of intensity-dependent coupling functional.
- Detuning parameters affect the maximum values of the atomic entropy by bringing them down, elongating the revival time and increasing the collapse period.
- The effect of the Kerr medium changes the quasi period of the field entropy evolution and add some regularity to the behavior of the atomic entropy.
- The Kerr-like medium and detuning have similar effects on the degree of entanglement, where they decrease the maximum values of the entanglement. However, they have opposite effects, where the detuning effect increases the revival period the Kerr effect decreases it.
- At very strong nonlinear interaction of the Kerr-like medium with the field mode, results in that the field and the atom are almost decoupled, hence pure states occur.

- The maximum values of the atomic entropy in the binomial field are larger than that for the squeezed coherent case for all considered cases except for the case in which the intensity-dependent coupling functional is taken as $f_i(n) = \frac{1}{\sqrt{n+1}}$.

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